Roger S. Bagnall/Alexander Jones (eds.): Mathematics, Metrology, and Model Contracts. A Codex from Late Antique Business Education (*P.Math.*). New York: New York University Press 2019. VII, 193 p., XXIV pages of plates. $ 85.00. ISBN: 978-1-4798-0176-3.

The book under review (MMMC henceforth) contains an edition of a fourth-century, one-quire notebook (called *P.Math.*) comprising in more or less random order the following items, most of which of obvious mathematical content: three model contracts (two of them for loan of money), five metrological lists (= units of measurement of length, area, volume, or liquid capacity), 31 standard problems of geometric metrology (= measurement of areas [14 items] and volumes [17] of geometrical figures or of objects measured as such), six problems of partition of common fractions into unit fractions, seven miscellaneous problems of mathematics (fictitiously) applied to everyday life, traditionally – and misleadingly – categorized as "recreational mathematics".[1] *P.Math.* is the richest document of this kind after the Akhmīn mathematical papyrus (*P.Cair. cat.* 10758).[2]

The Introduction of MMMC presents all elements required to understand the context and contents of *P.Math.*: the history of its discovery (a thrilling [hi]story, as usual in this field), its codicological features, its script and its date, the characteristics of the Greek language in which the texts are written, a detailed discussion of the technical background to the several kinds of problems (metrology, measurement of geometric figures, partitions into unit fractions), nature and purpose of *P.Math.*, a list and a typology of the problems. The introduction is followed by the edition, with facing translation, and by a problem-by-problem commentary. A complete set of indices is

1 Collections of such problems in Greek and non-Greek sources can be found in K. Vogel (ed.): Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts. Text, Übersetzung und Kommentar. Wien/Graz/Köln 1968 (Wiener byzantinistische Studien 6), 154–160; H. Hunger/K. Vogel (eds.): Ein byzantinisches Rechenbuch des 15. Jahrhunderts. 100 Aufgaben aus dem Codex Vindobonensis Phil. Gr. 65. Text, Übersetzung und Kommentar. Wien/Graz/Köln 1963 (Österreichische Akademie der Wissenschaften. Denkschriften. Philosophisch-historische Klasse 78.2), 91–101; and, on a systematic basis and ranging over the entire worldwide corpus, in J. Tropfke: Geschichte der Elementarmathematik.Vollständig neu bearb. v. K. Vogel. Bd. 1: Arithmetik und Algebra. 4. Aufl. Berlin/New York 1980, sect. 4.

2 J. Baillet: Le papyrus mathématique d'Akhmîm. In: Mémoires publiés par les membres de la Mission Archéologique Française au Caire 9,1, 1892, 1–89.

preceded by two appendices, which list edited papyri of the same kind as *P.Math.* and give a synopsis of their contents, and by the bibliography.

MMMC is an important publication and provides a wide range of information; readers and scholars can acquire a complete and in-depth knowledge of this specific subfield of papyrology by using MMMC as a reference. In my personal perspective, MMMC is also important because it makes explicit with uncompromising force the two basic assumptions that underlie – as a careful reading of all editions of mathematical papyri confirms – the prevailing approach of modern scholarship to this kind of texts: that this is trivial mathematics and (therefore) that it can only be a school product. These two assumptions I would like to question briefly here.

As for the first assumption, I have to say that MMMC's proclivity for adjectives like "trivial", "contrived", "roundabout", "absurd", "superfluous", "disastrous", "approximate" (even when no exact algorithm could exist) is in the end disturbing. The algorithm for computing the area of a rectangle is not "trivial": it is simply what is required. What might be questioned is the inclusion of the area of the rectangle in the list. But well, a fairly complete vade mecum of computational lore may well – and I would rather say: should – include a problem that explains how to find the area of a rectangle. After all, we find this problem in Hero's *Metrica* and in all compilations of geometric metrology that medieval manuscripts have handed down to us. Another example of MMMC's censorious attitude is the discussion of the "Surveyor's formula" (at 28), namely, estimating the area of a quadrilateral by taking the arithmetic mean of the opposite sides and by multiplying the results by each other; this prescription has been used for a millenary in several civilizations to the apparent satisfaction of all the actors involved. Drifting away from rectangular figures, for which the algorithm is exact, MMMC's evaluation of the estimates the algorithm provides shift in a climax from a "meaningful approximation" to an "overestimate" and to an "extreme" error, which ends in "mak[ing] nonsense" of a specific problem. Well, the nonsense simply lies in the fact that the figure of the nonsense problem is not a quadrilateral but a straight line: every algorithm for measuring the area of a trapezium would produce nonsense (a zero area is an excellent approximation of nonsense, and not only to Greek eyes) if applied to a straight line.

As for the school context, the line of argument is as follows: (trivial) mathematics + debased Greek language (misspellings, aberrant morphology, bad syntax) = teaching at school; if, as in this case, the spellings are a real mess and grammar and syntax fare just a bit better, the scribe can only be a student and not a teacher; but if, as in this case, the handwriting is rather fluid and well-shaped, and moreover the contents are after all not so trivial and the computations are correct even if they are "often applied in the service of mistaken algorithmic procedures", this student can only be a "student in a school devoted to training business agents" (at 55), that is, a young semi-analphabetic engaged in learning technical matters. This conclusion is reached by successive approximations in the course of the entire Introduction, but one fact is clear from the very beginning: the school context is assumed on the mere grounds that a similar assumption (albeit seldom corroborated by direct evidence) has been made in all previous editions of similar papyri and in reference studies on ancient literacy and numeracy. Well, it is time to state clearly that such an assumption – unless, let me repeat, corroborated by positive evidence – is totally unwarranted. Mine is not just a hypercritical stance: the fact is that all that we know of the production and transmission of similar documents in medieval Byzantium points to a wider range of possibilities. A possibility that very often obtains in Byzantium is that these products are (part of) notebooks written for personal use (maybe for professional use, maybe not) by a scholar, a monk, a teacher, a notary, a trader, a semi-literate adult or young adult: by anyone involved in any intellectual activity and for some reason interested in noting down more or less consistent and structured pieces of technical lore. And all these documents display the same features as we find in *P.Math.* and in so many mathematical papyri: trivial contents if compared to the heights of Greek mathematics or even to the comparatively lower heights of Byzantine mathematics; systematic use of the vernacular, spiced with curious misspellings and morphosyntactical aberrations; mathematical mistakes and redundancies of any kind; servile copying instead of real understanding of what is transcribed.[3] Nor can papyri and medieval manuscripts be differentiated on codicological grounds: recent research on Byzantine manuscript production has finally made it clear that most of what we now find collected under the cover of a

3 A case in point is Par. suppl. gr. 387: M.-L. Concasty: Un manuscrit scolaire (?) de mathématiques. In: Scriptorium 21, 1967, 284–288, and, for an analysis in this perspective, F. Acerbi: Struttura e concezione del vademecum computazionale Par. gr. 1670. In: Segno e Testo 19, 2021, in print.

codex was first produced as (sets of) separated quires and then assembled and reassembled for a variety of purposes.[4]

P.Math. can in fact be categorized as a little *Rechenbuch*,[5] namely, a collection of computational techniques and of arithmetical or geometric metrological problems fairly unrelated to each other, sometimes in (fictitious) daily-life guise, sometimes organized in sequences of almost identical items, and often formulated in a debased algorithmic code. The *Rechenbücher* are one of the most characteristic features of Medieval mathematics, both in the West and in the East, and *P.Math.* is a welcome corroboration of the view that these literary products find their root in Late Antiquity.

Rechenbücher, and the manuscripts containing them together with other pieces of mathematical lore such as Easter Computi and geometric metrological collections, constitute a continuous and extremely rich variation on the leitmotiv of computing with integers and fractions, both in general terms and in the framework of well-defined algorithms (and this also explains the highly schematic character of most diagrams that illustrate the problems). In this perspective must be read the most amazing (but in MMMC, to the first of these problems the adverb "absurdly" is attributed) couple of problems of the whole *P.Math*: they indisputably compute the volume content of a four-dimensional object "represented" as a vaulted granary; this can be compared with the geometric δυναμοδύναμις "power of power" in Hero's *Metrica* I.17. In this perspective must also be read the presence, together with the main bulk of geometric metrological problems, of metrological lists, of the partitions of common fractions into unit fractions, and of the riddles. The model contracts just give a personal touch to our one-quire *Rechenbuch*. Not at all trivial mathematics, not necessarily school exercises, quite possibly a vade mecum of someone engaged professionally in technical matters.[6]

4 See G. Cavallo: Stralci di storia di un gruppo di manoscritti greci del secolo IX. In: P. Chiesa/A.M. Fagnoni/R.E. Guglielmetti (eds.): Ingenio facilis. Per Giovanni Orlandi (1938–2007). Firenze 2017 (Millennio medievale 111), 3–64; F. Acerbi/A. Gioffreda: Manoscritti scientifici della prima età paleologa in scrittura arcaizzante. In: Scripta 12, 2019, 9–52: 9–10 and 26–34.

5 See F. Acerbi: Byzantine Rechenbücher: An Overview with an Edition of Anonymi L and J. In: JÖByz 69, 2019, 1–58.

6 For a parallel between multiplication tables and the libretti of Italian merchants see already J. Sesiano: A Reconstruction of Greek Multiplication Tables for Integers. In: Y. Dold-Samplonius/J.W. Dauben/M. Folkerts/B. van Dalen (eds.): From China to

For the reasons just outlined, the reader might have even more profited from MMMC had it offered systematic references to similar problems found in Hero's *Metrica* and in the pseudo-Heronian *corpus*, as well as to similar riddles and partitions into unit fractions in Byzantine *Rechenbücher*. Likewise, some items – admittedly just fragments – appear to be missing in the lists of the final Appendixes (where we read both "Achmin" and "Akhmin").[7]

The section on "Algorithms used in problems" is interesting and deserves a couple of remarks. The algorithms are there first described by formulating the independent steps in full words and by listing them in succession, and then by providing a transcription of them as a sequence of equalities formulated in strictly symbolic notation. This transcription I find not adequate, and for two reasons. First, inconsistency of the mapping between formulation in full words and the sequence of equalities: the latter is sometimes missing (P4ib), sometimes subsumes all or many steps in one single formula (*passim*), sometimes contains spurious steps (e.g., the last equality in both P10A and P10Ai). Second, if symbolic transcriptions have to be used – and I think they must – they should be faithful to the algorithm, not to a complete "formula" that never appears in the text. Thus, the algorithm of problem P4 could for instance be transcribed as follows:[8]

$$(a,b,c,d) \rightarrow a + b \rightarrow (a + b)/2. \quad c + d \rightarrow (c + d)/2 \rightarrow [(c + d)/2][(a + b)/2] = A.$$

The full stop in the middle of the sequence marks a hiatus in the algorithm, namely, a step that does not accept the output of the immediately preceding step as input. (See below for punctuating an algorithm in the Greek text.)

The edition is correct and proposes the most plausible readings for the frequent (albeit small) lacunae in the papyrus (but I suggest reading a form of

Paris: 2000 Years Transmission of Mathematical Ideas. Stuttgart 2002 (Boethius 46), 45–56: 54.

7 Cf. the list of geometric metrological papyri in F. Acerbi/B. Vitrac (eds.): Héron d'Alexandrie, Metrica. Introduction, texte critique, traduction française et notes de commentaire. Pisa/Roma 2014 (Mathematica Graeca Antiqua 4), 557–570, where, however, some of the items in MMMC, Appendix 2, are missing. Lists of tables of partitions of common fractions into unit fractions are given in D.H. Fowler: A Catalogue of Tables. In: ZPE 75, 1988, 273–280, and D.H. Fowler: Further Arithmetical Tables. In: ZPE 105, 1995, 225–228.

8 Cf. Acerbi: Byzantine Rechenbücher (see n. 5), 20–21 for my conventions, and passim for examples.

δαπανᾶν at 66.14). The critical apparatus almost entirely comprises corrections of misspellings and incorrect morphs, whose original forms can be found in the main text. On the one hand, this is the standard practice. On the other hand, one might wonder whether correcting all these trivialities – thereby obscuring more interesting variant readings that might figure in the apparatus – is really necessary: after all, the readership of an edition of a technical papyrus with facing translation should not find difficulties in understanding that ἥμυσυ stands for ἥμισυ. A list of the most frequent misspellings could be provided just before the edition.

Following rules that I have explained elsewhere,[9] I would punctuate the algorithms in the Greek text according to a criterion different from simply putting full stops at every step, a choice that flattens the operational structure of the algorithms. For instance, at 86.2–9, I suggest punctuating as follows:

εὑρεῖν τὰς ἀρούρα[ς.]
[οὕ]τω ποιοῦμεν. συντίθω τὸ νώτον καὶ τὸν βορέα,
[η] καὶ ς· γί(νεται) ιδ· ὧν ἥμισυ ζ· ἐπὶ τὸν β (διὰ τί ἐπὶ τὸν β; [ὅτι]
τὸ βῆμα ἔχει πήχεις β)· γί(νεται) ιδ πήχεις. καὶ συντίθω
[τ]ὸν ἀπηλιώτον καὶ τὸν λιβάν, ιε καὶ ιγ· γί(νεται) κη· ὧν
[ἥ]μισυ ιδ· ἐπὶ τοῦ πήχεις· γί(νεται) πδ· πδ ἐπὶ τὸν ιδ·
γί(νεται) [Ἀ]ρ[ο]ς· λοιπαὶ Ἀρος· ⟨παρὰ⟩ τὸ ʽΘσις· γί(νεται) τπδ.
[ἔσ]ται ἄρουραι τπδ. οὕτως ἔχει ὁμοίως.

An upper point links two steps whenever the second step accepts the output of the first as input, and separates the statement of the result of an operation from the operation itself; a full stop marks a hiatus; metamathematical remarks (a stylistic trait, not a sign of teaching) are singled out by parentheses or dashes.

The mathematical commentary contains some inconsistencies or infelicities of formulation: “right triangle” along with “right-angled triangle” (e.g. at 31; the second expression should be used); 32.3 “difference” should read “product”; “vertical” along with “altitude” along with “height” (*passim*; the third should be used); some algorithms are termed “approximate” misleadingly (the Surveyor’s formula, that cannot be so termed) or uselessly (from 38 on; *every* algorithm for round figures is approximate if it involves computing the

9 See in the first place F. Acerbi: I codici stilistici della matematica greca: dimostrazioni, procedure, algoritmi. In: QUCC 101, 2012, 167–214: 213; see also Acerbi/Vitrac: Héron d’Alexandrie (see n. 7), 98.

area of a circle); “similar equilateral triangles” (at 45–47; all equilateral triangles are similar); at 49 the symbolic transcription of the algorithm contains a spurious factor 13/30. In the translation, I would render multiplicative ἐπί with “by”, for “times” is better used for multiplications expressed by -ακις adverbs.

MMMC would have benefited from a more careful proofreading. I have found the following typos in the Greek texts: 64.15 ἕσται → ἔσται; 66.5 ὅσαδί[→ ὁσαδί[; 76.15 last word read *stigma* instead of final *sigma*; 80.4 πῆχυς ἐστὶν → πῆχύς ἐστιν; 88.5 ἥ → ἣ; 92.16 ἕχει → ἔχει; 94.13 ἄρουρα ἐστὶν → ἄρουρά ἐστιν; 98.4 add full stop after ἥμησυ; 100.1 χώρη[σει → χωρή[σει; 106.23 προσθὲς → πρόσθες; καί εἰσὶ → καί εἰσιν; 114.3 ὤστε → ὥστε; 114.22 ἦν → ἣν (?); 124 ἐπειδὴ → ἐπειδή; 157.8 lines from bottom ὑημσίας → ὑμησίας; 190 last line τέσσάρες → τέσσαρες. References to “section 10” throughout MMMC should read “section 11”. I also failed to make sense of the indication “cf. Gr. 6” in the apparatus to 94.5.

An excessively punctilious reader will also notice that five different Greek fonts have been used: two of them are systematically mixed (also in the edition: check 62.14–20); the other three fonts are used just once each, in the next-to-last line of page 136 and in the first line of page 186, respectively, and three times, at 192, entries Μεχείρ, ἀπό, and ἐμαυτοῦ. The same punctilious reader will also remark that an excessively large font size has been used throughout MMMC; this makes for instance the tables of the two appendices unnecessarily difficult to use, apart from being an aesthetically unsatisfactory choice. Likewise, the figures in the introduction are too big and for this reason inelegant.

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Empfohlene Zitierweise

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